Multi-Dimensional Instructional Frameworks: A Promising Form of Domain-Specific Instructional Design

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Literature on learning trajectories is reviewed, and challenges in developing designs to support responsive instruction are highlighted. A distinctive form of design called a *multi-dimensional instructional framework* is introduced. These frameworks span key dimensions of mathematisation, such as notating and distancing the setting, with each dimension outlined by a progression of levels. A framework comprises the matrix of combinations of levels across its dimensions. The framework form is described, and illustrated with the frameworks for two domains, drawn from a program of design research. The potential of the form to support responsive instruction is discussed, and considered to be promising.

In the endeavour of instructional design, Confrey (2006) advised: "One cannot prescribe practices, but one can guide practice by means of explanatory frameworks accompanied by data, evidence, and argument" (p. 139). Researchers and instructional designers have sought to develop forms of explanatory frameworks that can effectively guide practice. Over the last 20 years, various forms described as learning trajectories and related terms have been designed, usually for specific domains of mathematical instruction such as early addition and subtraction or multiplicative thinking.

In a program of design research (Cobb & Gravemeijer, 2008) on intervention in number learning (Wright & Ellemor-Collins, 2018), we have developed a form we call a *multi-dimensional instructional framework*. The aim of this paper is to describe the form and its potential contribution to instructional design. The paper is presented in four sections. Firstly, we briefly review forms of instructional design, and the kind of instruction we aspire to support with our designs. Secondly, we acknowledge our research program as a basis for our designs. Thirdly, we describe our proposed form, the multi-dimensional instructional framework, drawing on examples from our design studies. Finally, we discuss some of the features and potential of the form.

Background: Forms of Instructional Design

Responsive, Inquiry-Based Instruction

A broad consensus of researchers aspires to a responsive, inquiry-based approach to instruction (e.g., Mason & Johnston-Wilder, 2006; van den Heuvel-Panhuizen, 2001). Teachers need to pose tasks that are genuine problems for students, and to pay attention to students' reasoning in responding to those tasks. Instruction needs to support *progressive mathematisation* (Freudenthal, 1991; Gravemeijer, Cobb, Bowers, & Whitenack, 2000), developing from students' informal, context-bound strategies toward more formal and sophisticated reasoning. Teachers need to keep adjusting instruction to the cutting edge of students' current knowledge, to provoke new insights and developments (Mason & Johnston-Wilder, 2006; Wright, Ellemor-Collins & Tabor, 2012). Given these aspirations, instructional designers seek to develop designs that can support responsive, inquiry-based instruction.

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Learning Trajectories and Learning Progressions

Learning trajectory has become a widely-used term for an instructional design drawing together research on learning and teaching. As an example, Clements and Sarama (2009) described their learning trajectories having three parts: “a specific mathematical goal, a developmental path along which children develop to reach that goal, and a set of instructional activities that help children move along that path” (p. ix). Many researchers promote learning trajectories as forms of design that can support responsive instruction (Empson, 2011; Sztajn, Confrey, Wilson, & Edgington, 2012), with Daro, Mosher, and Corcoran (2011) concluding that they “hold great promise as tools for improving instruction in mathematics” (p. 13).

As designers seek to best support instruction, a variety of related forms of design have been developed (Battista, 2011). When Simon (1995) coined the term *hypothetical learning trajectory*, he was describing a teacher’s local instructional planning for a class: the imagined sequence of tasks she might pose, what responses she anticipates, and what conceptual challenges the students might be working with. By contrast, Clements and Sarama’s (2009) 10 learning trajectories were developed from large-scale research projects, and presented as extended tables listing summaries of each level in a developmental progression over several years, with corresponding instructional tasks. Siemon and colleagues’ (2018) *learning progressions* have a similar structure of developmental levels, but were derived using Rasch modelling. Dutch teams developed another form, *learning–teaching trajectories* (van den Heuvel-Panhuizen, 2001), presented in textual descriptions organised in various phases and topics which overview the learning process and how didactics interact with the learning.

Challenges for Forms of Instructional Design

Aspiring to support responsive instruction presents challenges to forming an instructional design. One challenge is that inquiry-based learning is not linear. For example, in Empson’s (2011) research using learning progressions, she found that deviations from the planned progression were “consistent and numerous” (p. 580), and concluded “trying to represent research on learning in terms of trajectories quickly gets complicated” (p. 577). Fosnot and Dolk (2001) opt for a non-linear design they call a “landscape of learning”; others use networks, or rich descriptions (Sztajn et al., 2012). Can we develop forms of instructional design that can hold the many convoluted paths of student learning, while remaining sufficiently clear to support the teacher?

A second challenge is grain size. Siemon and colleagues (2018) try to meet teachers’ needs by providing useful summaries of zones of learning through major domains of high school mathematics, but admit that “the risk is that the grain size is large” (p. 46). Battista (2011) asks that an instructional design have “indications of jumps in sophistication that are small enough to fall within students’ ‘zones of construction’” (p. 530), to adjust to their cutting edge. For some, these can be small jumps indeed (Wright & Ellemor-Collins, 2018). Extending his vision, Battista asks if a design can support not just planning, but moment-to-moment teaching, to “continuously adjust instruction to meet students’ evolving learning needs” (p. 513). We are interested in forms of design that meet these challenges.

Research Program: Developing Pedagogical Tools for Intervention

Wright has led a long-running program of research and development since the 1990s, addressing intervention in number and arithmetic with low-attaining primary students (Wright & Ellemor-Collins, 2018; Wright, Ellemor-Collins & Lewis, 2007; Wright et al., 2012). The program has included several multi-year projects, involving intensive professional development with groups of teachers and intervention with students. The program uses a design research methodology (Cobb & Gravemeijer, 2008), involving cycles of assessment and teaching recorded on video, and analysed to better understand and refine the instructional designs. The research has focused on developing pedagogical tools, such as schedules of assessment tasks, models of learning progressions, and instructional frameworks, along with associated instructional theory. Studies have addressed designs for specific domains of number, such as multi-digit addition and subtraction (Ellemor-Collins & Wright, 2011a) and multiplicative computation (Ellemor-Collins, 2018). The designs discussed below draw on this body of research.

Multi-Dimensional Instructional Frameworks

We have developed a form of domain-specific instructional design we call a *multi-dimensional instructional framework*. We have designed multi-dimensional instructional frameworks for nine domains of number knowledge (Wright & Ellemor-Collins, 2018). We will describe this form of framework using an example: the framework for a domain we call Conceptual Place Value (CPV) (Ellemor-Collins & Wright, 2011a; Wright et al., 2012). CPV involves learning to flexibly increment and decrement numbers by 1s, 10s, and 100s, an informal knowledge of unitary coordination that is foundational for mental computation with multi-digit numbers.

An instructional framework for a given domain is organised around a few key *dimensions of mathematisation*. Each dimension involves a progression of levels. Figure 1 shows the CPV framework comprising three dimensions: extending the range of numbers, distancing the setting, and making the increments more complex. The progression of levels in each dimension is also outlined in the figure.

The framework then forms the matrix of all possible combinations of levels across these dimensions. To present this matrix of possibilities, we construct a chart in rows and columns, somewhat like a Cartesian product space. Figure 2 shows a portion of the CPV chart (Wright & Ellemor-Collins, 2018). Without needing to explain all the domain-specific terms in this abbreviated chart, the figure is sufficient to illustrate how such a chart can work.

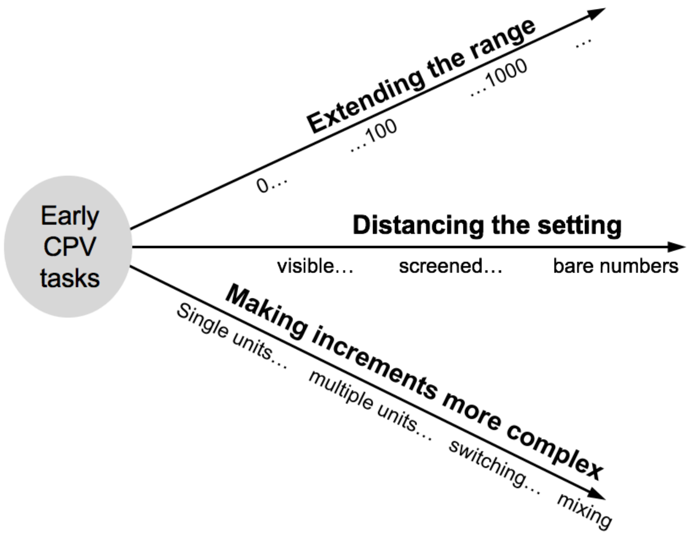

Figure 1. Three dimensions of mathematisation in Conceptual Place Value.

Topic	Teaching procedures			
RANGE I: 0 to 130				
Inc- & decrementing by 10s	*Bundling sticks shown* Say each number	*Bundling sticks screened* Say each number		
Inc- & decrementing by 1s and 10s	*Bundling sticks shown* Extend to multiple, switched, & mixed units	*Bundling sticks screened* Extend to multiple, switched, & mixed units	*Sticks + arrow cards* Say numbers, and build with arrow cards	…
RANGE II: 0 to 1000				
Inc- & decrementing by 10s	*Dot materials shown* Say each number.	*Dot materials screened* Say each number.		
Inc- & decrementing by 1s, 10s, & 100s	*Dot materials shown* Extend to multiple, switched, & mixed units	*Dot materials screened* Extend to multiple, switched, & mixed units	*Dots + arrow cards* Say numbers, and build with arrow cards	…
RANGE III: 0 to 1000 and BEYOND				
Inc- & decrementing by 1s, 10s, & 100s	*Dot materials shown* …	*Dot materials screened* …	*Dots + arrow cards* …	…
…				

Figure 2. Partial chart of the instructional framework for Conceptual Place Value.

The dimension of *extending the range* is laid out in big steps running vertically down the chart, progressing from Range I to Range III. The dimension of *making the increments more complex* is nested within each of those ranges, from incrementing by 10s only to incrementing by 1s and 10s; and is further indicated within some of the cells to extend to switched, multiple, and mixed units. The dimension of *distancing the setting* is laid out horizontally along each row, from materials shown, to screened, to introducing arrow cards. With the three dimensions thus spanning the rows and columns of the chart, each white cell of the chart indicates a combination of calibrations on those three dimensions. For example, the top left cell indicates instruction with range low (to 130), complexity low (incrementing by single 10s), and setting low (sticks shown). Importantly, the intention is not that instruction will progress through every cell, nor that it will progress systematically through each row and column. Rather, the framework provides a schematic map of possibilities for instruction that can adjust forwards or backwards along any of the dimensions at any point.

For example, early CPV instruction will typically stay in the lowest range, and advance along the dimension of complexity by incrementing multiple 10s, switching between increments of 10s and 1s, and incrementing by mixed 10s and 1s. Alongside these advances in complexity, the setting dimension will move between visible and screened: After some initial visible tasks, the setting is screened to challenge the students' visualisation, but the screen is also lifted sometimes for the students to check their answers and discuss their reasoning. On the chart, this instruction works around the square of four cells in the top left.

As fluency is established with these tasks, one line of instructional progression—suggested moving across the second row of the chart—advances the setting dimension further by introducing numerals, and moving towards tasks in bare numbers. For this progression, the range remains low, and the complexity might be retreated to simple increments while the written setting is introduced, before advancing again to switched and mixed increments. Another line of progression—suggested moving down the first column—maintains the setting in the lower levels of visible and screened materials, but extends the range towards 1000, introducing materials for 100s as well as 10s and 1s. This higher range will involve a new progression in the complexity of the increments: introducing simple increments of 100, then developing towards switching and mixing of three different units. The setting will again move back and forth between visible and screened materials. Later

CPV instruction progresses to advanced levels on all three dimensions, which would appear in the lower right cells of the chart, with tasks in the range across and beyond 1000, increments mixed and complex, and a setting of bare numbers.

While these are common lines of development, many idiosyncratic paths are possible. The framework does not direct a sequence of tasks. Rather, it lays out a map of potential progressions and adjustments, to support the teacher to make small adjustments in tasks that advance or retreat the level of challenge, along different dimensions, and with different combinations of dimensions, all leading toward significant mathematisation in the domain.

Dimensions of Mathematisation

Central to the construction of these instructional frameworks is the notion of a dimension of mathematisation (Ellemor-Collins & Wright, 2011b; Wright & Ellemor-Collins, 2018). The notion builds on research identifying particular forms of mathematising prominent in instruction, such as decimalising (e.g., Freudenthal, 1991); symbolising (e.g., Gravemeijer et al., 2000); and generalising (e.g., Mason & Johnston-Wilder, 2006). Dimensions of mathematisation are also related to the notion of *dimensions-of-possible-variation* in tasks (Mason & Johnston-Wilder, 2006). However, when describing longer progressions in instruction, the dimensions extend further than individual tasks: they become the key progressions within a whole instructional domain. Ellemor-Collins and Wright (2011b) developed a list of 10 dimensions of mathematisation significant across many domains of arithmetic instruction, including: distancing the setting, formalising, generalising, notating, refining computation strategies, structuring numbers, and unitising numbers.

Describing a Multi-Dimensional Instructional Framework

The most concise description of an instructional framework identifies the key dimensions, and the levels in each dimension, as in Figure 1. The teaching chart, as in Figure 2, offers more guidance, by including instructional tasks across the whole matrix of possible combinations of levels, and suggesting basic lines of progression through this matrix. Ultimately, a full description of a framework requires even more detail, explaining task types, the use of settings, subtler adjustments along the dimensions, and relationships between the dimensions, alongside accounts of student responses and progressions (Wright et al., 2012). Video exemplars of instruction and analyses of student learning can contribute to a rich description of the instructional design.

A Second Example: The Instructional Framework for Multiplicative Basic Facts

A second example of a multi-dimensional instructional framework is the framework for the domain of Multiplicative Basic Facts (Ellemor-Collins, 2018; Wright & Ellemor-Collins, 2018). The domain develops multiplicative strategies for mental computation of multiplication and division, and increasing automaticity with the basic facts. It is a more complex domain than CPV, and the framework comprises, not three, but five key dimensions. Figure 3 is a schematic chart for the framework, showing the Range dimension (RNG) progressing vertically; the Structuring and strategies dimension (STR) nested within each range; the Setting (SET) and Notation (NTN) dimensions overlapping horizontally; and the Orientation dimension (ORN) varying within each cell. The richer description of the framework includes, for example, peculiarities within each range, the intertwining of the setting and notation dimensions, and ways the varying orientation interacts with the setting.

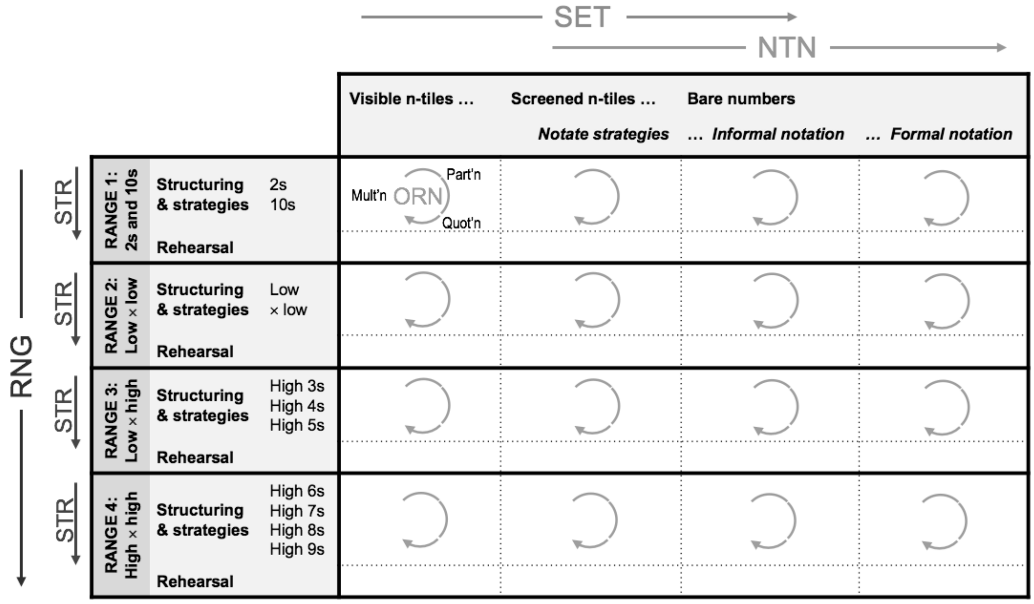

Figure 3. Schematic chart for the instructional framework in Multiplicative Basic Facts, showing the span of five dimensions: Range (RNG), Structuring (STR), Setting (SET), Notation (NTN), and Orientation (ORN).

Also, more fine-grained progressions are described for each of the dimensions. For example, gradations in the setting dimension include: partially screening materials, flashing materials, requesting descriptions of the materials when screened, and referring to materials without using them. A subtle gradation in the range is shifting from even to odd multiples of 5.

Discussion

Earlier we described challenges to develop forms of instructional design that support responsive, inquiry-based instruction. We propose that the multi-dimensional instructional framework is a promising form. It creates a wide space of instructional tasks in which to observe students' problem-solving, and to engage in dialogue with them about their mathematical activity (Mason & Johnston-Wilder, 2006). The framework does not direct the teacher on a pre-determined sequence; rather, it offers options for the teacher to respond to the dialogue by adjusting instruction forwards, sideways and backwards.

To support progressive mathematisation, a multi-dimensional framework is based on key dimensions of mathematisation, and orients the instruction to progress along these dimensions. Treffers (1987) suggested that "besides macro-levels in the learning process, one can distinguish finer meshes and a stepwise structure of mathematising at the micro-level" (pp. 248–9). We find that a well-developed multi-dimensional framework can illuminate this micro-level, offering the teacher potential adjustments in these finer meshes.

To support teaching at the cutting edge, a multi-dimensional framework indicates fine-graded adjustments on multiple dimensions. The framework reveals how to adjust a single dimension only, forwards or backwards, and how a single task can be extended in a few different directions, without any one being a jump too far-the 'small jumps' Battista (2011) seeks. Furthermore, the framework indicates these possibilities at any point in the instructional map, so a teacher can not only pitch to the cutting edge, but can keep adjusting tasks along a changing cutting edge. The multi-dimensional form can be a powerful support for responsive instruction.

A multi-dimensional framework offers a space that can hold the rich detail of manifold convoluted paths. At the same time, the framework can crystallise what is common across those paths: progression along a handful of dimensions. The combined potential for simplicity and detail has proven a compelling characterisation of instruction, in our analyses of observed intervention, and in our professional development work with teachers.

Guiding Instruction with Dimensions rather than Phases

A common approach to instructional design is to identify phases of instruction. Most of the examples of learning trajectories reviewed earlier in the paper are constructed as a single sequence of phases or levels—the trajectories of Clements and Sarama (2009), and the progressions of Siemon and colleagues (2018) for instance. Multi-dimensional instructional frameworks are distinctive in being constructed as a span of dimensions, rather than a sequence of phases. To highlight the difference with a metaphor: When giving directions to the castle, instead of saying "through the marsh, through the forest, and across the field", we say "head northwards, and westwards, and uphill". Instead of identifying lateral phases that get travelled *across*, we identify longitudinal dimensions that get travelled *along*.

We have found this distinctive form illuminating. For example, in Ellemor-Collins's (2018) study of a sequence of lessons on multiplication, phases of instruction could be identified retrospectively: Phase A involved a setting of materials but no notation, while Phase B involved notation but no materials. An instructional design could duly recommend a teacher look out for when Phase A is ending, and transition to Phase B. However, such a phase-oriented design would not align with the actual instructional decision-making observed during those lessons. In the lessons, notation was introduced not because of student accomplishment with the materials, but because of student confusion, and an attempt to reconnect to the cutting edge of the student's understanding. Over the two lessons after the notation was introduced, the student attended increasingly to the notation and decreasingly to the setting, and the teacher responded to this shifting attention with adjustments in the tasks. Ellemor-Collins argued that the organisation of instruction be described "not as an attempt to transition between two phases, rather as a consistent attempt to keep [the student] at his cutting edge, strategically adjusting two dimensions to do so: the setting and the notation" (p. 309). From a phase perspective, those two lessons can get sidelined as transitional; but from a dimension perspective, the lessons appear significant, and they illuminate progressions in two dimensions and the interactions between those dimensions.

Further Research

We have done considerable design research developing multi-dimensional frameworks for domains of arithmetic, especially in the fine-graded detail useful for one-to-one intervention. Three immediate lines of research beckon. One is to modify the frameworks we have developed to suit the particular needs of classroom teachers. We have begun pilot studies in this development. A second is for researchers to develop multi-dimensional frameworks for other domains of mathematics instruction. A third is to study how teachers are learning and using these frameworks, to better understand how the frameworks work for teachers. We hope this promising form of design can be applied widely.

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References

Battista, M. T. (2011). Conceptualizations and issues related to learning progressions, learning trajectories, and levels of sophistication. *The Mathematics Enthusiast, 8*(3), 507-570.

Clements, D. H., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach.* New York, NY: Routledge.

Cobb, P., & Gravemeijer, K. (2008). Experimenting to support and understand learning processes. In A. E. Kelly, R. Lesh & J. Baek (Eds.), *Handbook of design research methods in education: Innovations in science, technology, engineering, and mathematics learning and teaching* (pp. 68–95). New York, NY: Routledge.

Confrey, J. (2006). The evolution of design studies as methodology. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 135-151). Cambridge, UK: Cambridge University Press.

Daro, P., Mosher, F., & Corcoran, T. (2011). *Learning trajectories in mathematics: A foundation for standards, curriculum, assessment and instruction* (CPRE Research Report #RR-68). Philadelphia, PA: Consortium for Policy Research in Education.

Ellemor-Collins, D. (2018). *Threads through a labyrinth: Characterising intervention instruction for multiplicative strategies as an interweaving of five dimensions of mathematisation.* (Doctoral dissertation), Southern Cross University, Lismore, NSW.

Ellemor-Collins, D., & Wright, R. J. (2011a). Developing conceptual place value: Instructional design for intensive intervention. *Australian Journal of Learning Difficulties, 16*(1), 41-63.

Ellemor-Collins, D., & Wright, R. J. (2011b). Unpacking mathematisation: An experimental framework for arithmetic instruction. In B. Ubuz (Ed.), *Proceedings of the 35th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 313-320). Ankara, Turkey: PME.

Empson, S. B. (2011). On the idea of learning trajectories: Promises and pitfalls. *The Mathematics Enthusiast, 8*(3), 571-596.

Fosnot, C. T., & Dolk, M. (2001). *Young mathematics at work: Constructing number sense, addition, and subtraction.* Portsmouth, NH: Heinemann.

Freudenthal, H. (1991). *Revisiting mathematics education*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Gravemeijer, K., Cobb, P., Bowers, J., & Whitenack, J. (2000). Symbolizing, modeling and instructional design. In P. Cobb, E. Yackel & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design* (pp. 225-273). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Mason, J., & Johnston-Wilder, S. (2006). *Designing and using mathematical tasks*. St Albans, UK: Tarquin Publications.

Siemon, D., Callingham, R., Day, L., Horne, M., Seah, R., Stephens, M., & Watson, J. (2018). From research to practice: The case of mathematical reasoning. In J. Hunter, P. Perger & L. Darragh (Eds.), *Making waves, opening spaces* (Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia) (pp. 40-49). Auckland, NZ: MERGA.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114-145.

Sztajn, P., Confrey, J., Wilson, P. H., & Edgington, C. (2012). Learning trajectory based instruction: Toward a theory of teaching. *Educational Researcher, 41*(5), 147-156.

Treffers, A. (1987). *Three dimensions: A model of goal and theory description in mathematics instruction--the Wiskobas Project.* Dordrecht, The Netherlands: D. Reidel Publishing Company.

van den Heuvel-Panhuizen, M. (Ed.). (2001). *Children learn mathematics: A learning-teaching trajectory with intermediate attainment targets for calculation with whole numbers in primary school.* Utrecht, The Netherlands: Freudenthal Institute, Utrecht University/SLO.

Wright, R. J., & Ellemor-Collins, D. (2018). *The Learning Framework in Number: Pedagogical tools for assessment and instruction.* London, UK: Sage.

Wright, R. J., Ellemor-Collins, D., & Lewis, G. (2007). Developing pedagogical tools for intervention: Approach, methodology, and an experimental framework. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia) (Vol. 2, pp. 843-852). Hobart: MERGA.

Wright, R. J., Ellemor-Collins, D., & Tabor, P. (2012). *Developing number knowledge: Assessment, teaching, and intervention with 7–11-year-olds*. London, UK: Sage.